Final Exam

24.01.2017

The time available for the exam is 3h. No calculators, books or slides are allowed, only one A4 handwritten paper with notes.
Write the correct answer (A,B,C,D or E) of each multiple-choice question in the following table. Write the final answers to the open questions in the provided space of each problem page.
Each question gives 1 point, as well as each step of the two open questions (total 30 points).

NAME..

N°SCIPER.........................

TABLE OF ANSWERS							
1	A	7	D	13	B	19	C
2	B	8	D	14	A	20	A
3	D	9	B	15	A	21	D
4	B	10	C	16	B	22	F
5	D	11	D	17	B	23	D
6	B	12	A	18	D	24	C

PHYSICAL CONSTANTS

$$q_e = 1.6 \times 10^{-19}\ C$$
$$m_e = 9.1 \times 10^{-31}\ kg$$
$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$$
$$k = (4\pi\epsilon_0)^{-1} = 8.9 \times 10^{9}\ Nm^2/C^2$$
$$\mu_0 = 4\pi \times 10^{-7}\ Vs/(Am)$$
$$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$
$$h = 6.63 \times 10^{-34}\ J \cdot s = 4.14 \times 10^{-15}\ eV \cdot s$$
$$c = 3 \times 10^{8}\ m/s$$

TRIGONOMETRY

$$\cos 30^\circ = \sin 60^\circ = \sqrt{3}/2$$
$$\cos 45^\circ = \sin 45^\circ = \sqrt{2}/2$$
$$\cos 60^\circ = \sin 30^\circ = 1/2$$

Question 1

A charge Q is distributed in a sphere of radius R with a volume charge density which varies linearly with the distance r from the center as $\rho = ar$. Evaluate the parameter a.

A. $\frac{Q}{\pi R^4}$

B. $\frac{3Q}{4\pi R^4}$

C. $\frac{3Q}{4\pi R^3}$

D. $\frac{Q}{\pi R^3}$

$$Q = \iiint \rho \, d\tau = 4\pi \int_0^R \rho \, r^2 \, dr$$

$$Q = 4\pi a \int_0^R r^3 \, dr = \pi a R^4$$

$$a = \frac{Q}{\pi R^4}$$

Question 2

Three positive charges $+q$ and three negative charges $-q$ are fixed on the corner of a regular hexagon of side a, as shown in the figure below. Evaluate the modulus of the electric field at the center of the hexagon.

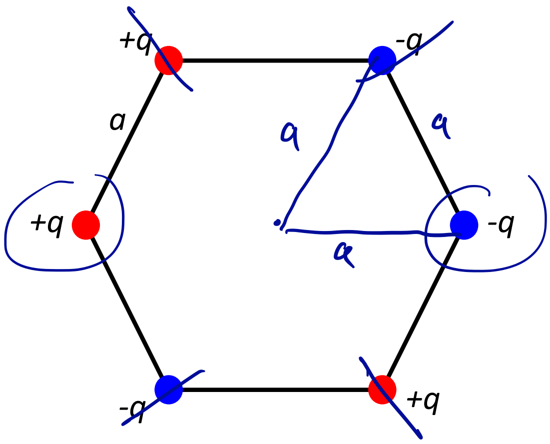
A. $\frac{q}{4\pi\epsilon_0 a^2}$

B. $\frac{q}{2\pi\epsilon_0 a^2}$

C. $\frac{3q}{4\pi\epsilon_0 a^2}$

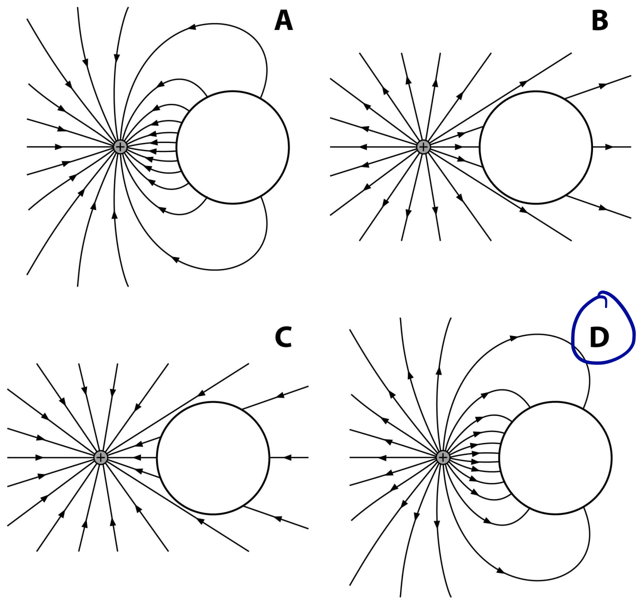
D. $\frac{3q}{2\pi\epsilon_0 a^2}$

$$E = 2 \cdot \frac{q}{4\pi\varepsilon_0 a^2} = \frac{q}{2\pi\varepsilon_0 a^2}$$



Question 3

A positive charge is placed next to a conductive sphere. Which of the following is the correct representation of the E field lines?



Question 4

Three positive charges $+q$ are placed on the corners of an equilateral triangle of side d. Evaluate the work done for bringing the charge in point A to point B in the middle of the opposite side, as shown in the figure below.

A. $\frac{q}{2\pi\epsilon_0 d^2}$

B. $\frac{q^2}{2\pi\epsilon_0 d}$

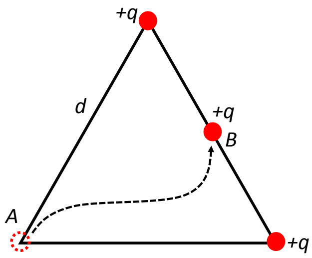
C. $\frac{q}{3\pi\epsilon_0 d^2}$

D. $\frac{q^2}{3\pi\epsilon_0 d}$

$$U_0 = 3 \cdot \frac{q^2}{4\pi\varepsilon_0 d}$$

$$U = \frac{2q^2}{4\pi\varepsilon_0 d/2} + \frac{q^2}{4\pi\varepsilon_0 d} = \frac{5q^2}{4\pi\varepsilon_0 d}$$

$$\Delta U = \frac{2q^2}{4\pi\varepsilon_0 d}$$

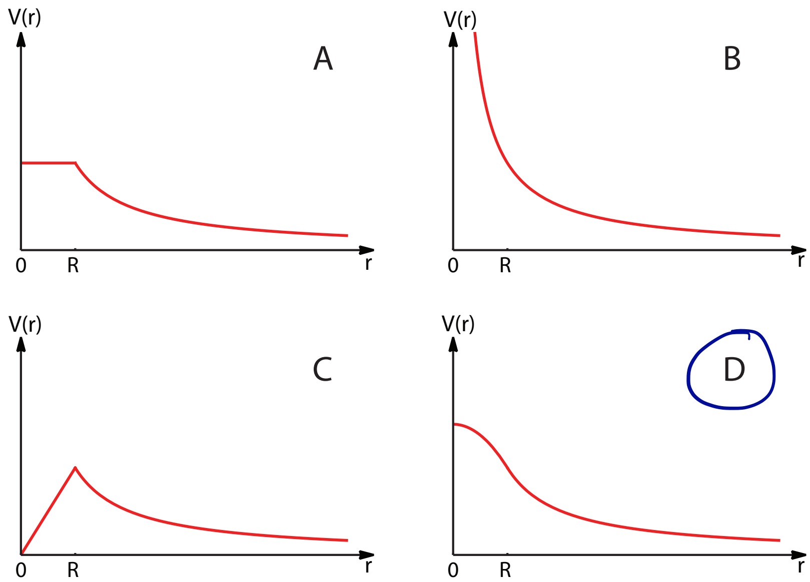


Question 5

A charge Q is homogeneously distributed inside the volume of a non conductive sphere of radius R. Which of the following is the correct plot of the electric potential as a function of distance from the center $V(r)$?

$r < R$

$E \propto r$



Question 6

Consider the electric field $\vec{E}_p$ of an electric dipole $\vec{p} = q\vec{\ell}$. Determine the ratio E_p/E_q of the modulus of the fields evaluated in the same point on the direction perpendicular to the dipole axis, where $\vec{E}_q$ is the electric field of a point charge q which replaces the dipole in its centre.

A. r/l

→ B. l/r

C. $2l/r$

D. $2r/l$

E. l

$$E_p = \frac{p}{4\pi\varepsilon_0 r^3} = \frac{q\ell}{4\pi\varepsilon r^3}$$

$$E_q = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$\frac{E_p}{E_q} = \frac{\ell}{r}$$

Question 7

A mixed beam of electrons e^- and positrons e^+ is sent with velocity $\vec{v} = v_0\hat{x}$ in a device of length ℓ with homogeneous electric field $\vec{E} = E_0\hat{y}$. Evaluate the separation d in the y direction at the end of the device between the position of the e^- and the position of the e^+, neglecting the interaction between them (a e^+ has same mass but opposite charge of an e^-).

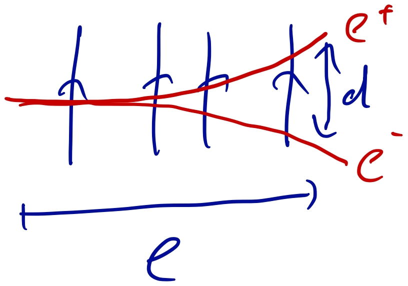
A. $\frac{eE_0\ell}{mv_0}$

B. $\frac{eE_0\ell}{2mv_0}$

C. $\frac{eE_0\ell^2}{2mv_0^2}$

D. $\frac{eE_0\ell^2}{mv_0^2}$

E. 0



time: $t = \frac{\ell}{v_0}$

$a_y = \frac{eE_0}{m}$

$\frac{1}{2}d = \frac{1}{2}a_y t^2 = \frac{1}{2}\frac{eE_0\ell^2}{mv_0^2}$

$d = \frac{eE_0\ell^2}{mv_0^2}$

Question 8

A positive charge is placed in the centre of two cubes, of which the length of the edge of the larger one is twice the smaller one. The flux of the electric field generated by the charge through one face of the small cube is ϕ_1. Evaluate the flux of the electric field through the upper half of the larger cube.

A. $\phi_1/2$

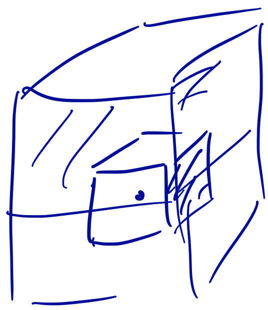
B. ϕ_1

C. $2\phi_1$

D. $3\phi_1$

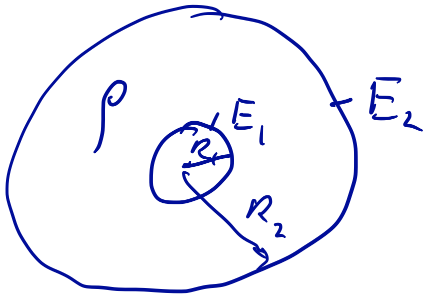
E. $4\phi_1$

$\Phi = \frac{Q}{\varepsilon_0}$



$\Phi_{tot} = 6\Phi_1$

$\frac{1}{2}\Phi_{tot} = 3\Phi_1$



Question 9

Because of the free charges in the Earth, an electric field E_1 is measured at sea level oriented perpendicular to the surface. An electric field E_2 is measured at the edge of the ionosphere along the same direction of E_1. Knowing the radius of the Earth R_1 and the radius of the ionosphere R_2 measured from the center of the Earth, evaluate the volume charge density ρ of all the atmosphere assuming that it is homogeneous.

A. $\frac{2\epsilon_0(E_2R_2 - E_1R_1)}{(R_2^2 - R_1^2)}$

B. $\frac{3\epsilon_0(E_2R_2^2 - E_1R_1^2)}{(R_2^3 - R_1^3)}$

C. $\frac{6\epsilon_0(E_1R_2^2 - E_2R_1^2)}{(R_2^3 - R_1^3)}$

D. $\frac{4\epsilon_0(E_2^2R_2 - E_1^2R_1)}{(R_2^2 - R_1^2)}$

$$4\pi R_1^2 E_1 = \frac{Q_E}{\varepsilon_0}$$

$$4\pi R_2^2 E_2 = \frac{Q_E}{\varepsilon_0} + \frac{4}{3}\pi(R_2^3 - R_1^3)\rho/\varepsilon_0$$

$$4\pi R_2^2 E_2 = 4\pi R_1^2 E_1 + \frac{4}{3}\pi(R_2^3 - R_1^3)\rho/\varepsilon_0$$

$$\frac{1}{3}\rho(R_2^3 - R_1^3)\frac{1}{\varepsilon_0} = R_2^2 E_2 - R_1^2 E_1$$

$$\rho = \frac{3\varepsilon_0(R_2^2 E_2 - R_1^2 E_1)}{R_2^3 - R_1^3}$$

Question 10

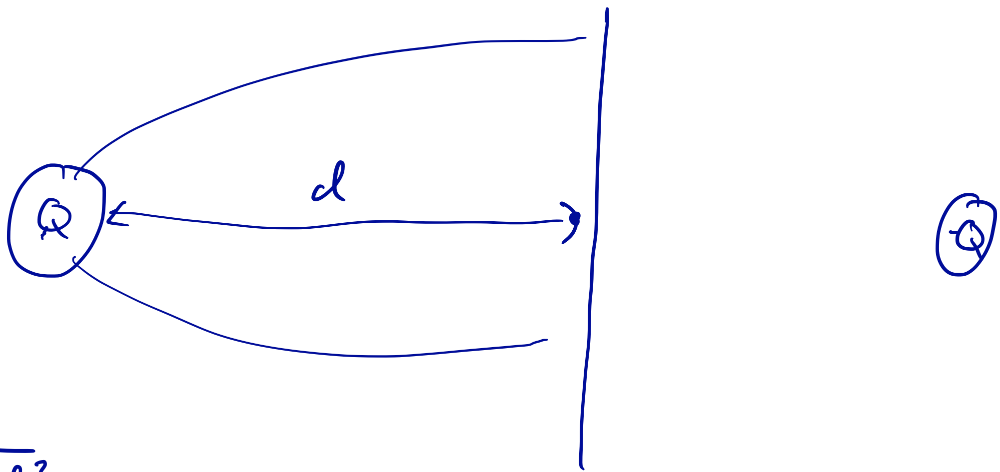
A charge Q is homogeneously distributed inside a non-conductive sphere of radius R. The center of the sphere is placed at distance $d >> R$ from a large conductive grounded plane. Evaluate the modulus of the E field at a point closest to the sphere at an infinitesimal distance from the plane.

A. $\frac{Q(d-R)}{2\pi\epsilon_0 d^3}$

B. $\frac{Q(d-R)}{4\pi\epsilon_0 d^3}$

C. $\frac{Q}{2\pi\epsilon_0 d^2}$

D. $\frac{Q}{4\pi\epsilon_0 d^2}$



$$E = 2\frac{Q}{4\pi\varepsilon_0 d^2}$$

Question 11

A soap bubble of radius R_1 is charged with a voltage V_1. When the bubble bursts, it collapses in a spherical droplet of radius $R_2 = R_1/10$. Evaluate the voltage of the droplet V_2.

A. $V_1/100$

B. $V_1/10$

C. V_1

D. $10\,V_1$

E. $100\,V_1$

Q conserved

$$C_1 V_1 = C_2 V_2$$

$$R_1 V_1 = \frac{R_1}{10} V_2$$

$$V_2 = 10 V_1$$

Question 12

Consider the four capacitors connected as shown in the figure below. Their capacitance is: $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$, $C_3 = 5\ \mu F$, $C_4 = 10\ \mu F$. A voltage difference $\Delta V_{\text{AC}} = 30\ V$ is applied between the points A and C. Evaluate ΔV_{BD}.

A. $10\ V$

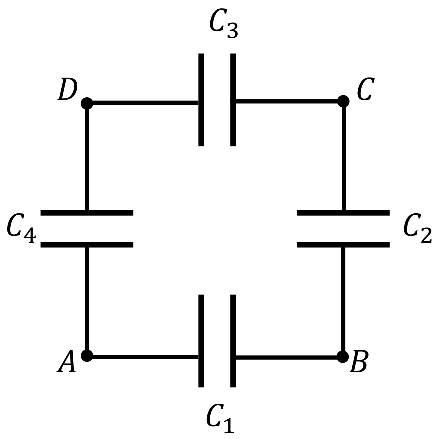
B. $15\ V$

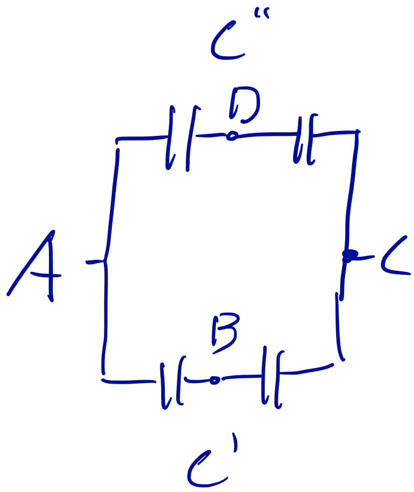
C. $20\ V$

D. $25\ V$

$$\frac{1}{C'} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \qquad C' = \frac{4}{3}$$

$$\frac{1}{C''} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10} \qquad C'' = \frac{10}{3}$$





$$Q' = C' V_{AC} = 40$$

$$Q'' = C'' V_{AC} = 100$$

$$V_{AB} = \frac{Q'}{C_1} = 20\,V$$

$$V_{AD} = \frac{Q''}{C_4} = 10\,V$$

$$\Delta V_{BD} = 10\,V$$

Question 13

A parallel plate capacitor of area A and distance d is charged with charge Q_0 and is kept operating at *constant voltage*. The space between the two plates is afterwards filled by a dielectric material of relative permittivity ϵ_r. Evaluate the charge that is now present on the plates.

A. Q_0 $\qquad Q = V_0 C = Q_0 \frac{C}{C_0} = Q_0 \epsilon_r$

B. $Q_0 \epsilon_r$

C. Q_0/ϵ_r

D. 0

Question 14

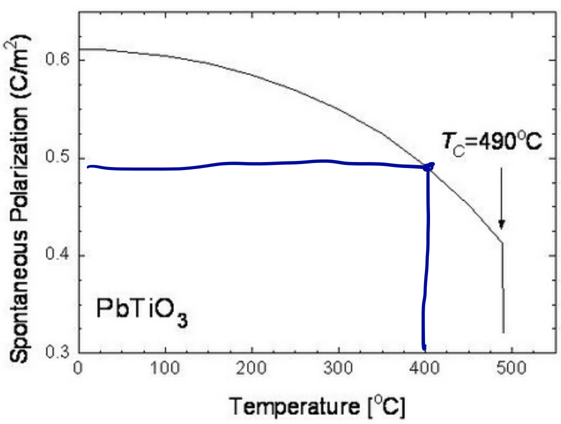
The electric polarization of $PbTiO_3$ as a function of temperature is shown in the figure below. The unit cell of the crystal is a cube of side 0.4 nm. Approximate the electric dipole moment associated to the unit cell at a temperature of 400° C.

A. $32 * 10^{-30}\ Cm$ $\qquad d\vec{p} = \vec{P}\, d\tau$

B. $32 * 10^{-27}\ Cm$ $\qquad = 0.5 (0.4)^3 \times 10^{-27}$

C. $128 * 10^{-30}\ Cm$

D. $128 * 10^{-27}\ Cm$ $\qquad = 32 \times 10^{-30}$



Question 15

A current I is passed through a wire made of four segments of length ℓ shaped as shown in the figure below. Evaluate the force acting on the wire placed in a homogeneous magnetic field $\vec{B} = B_0\hat{x}$.

A. $(1 - \frac{\sqrt{2}}{2})I\ell B_0\hat{z}$

B. $(\frac{\sqrt{2}}{2} - 1)I\ell B_0\hat{z}$

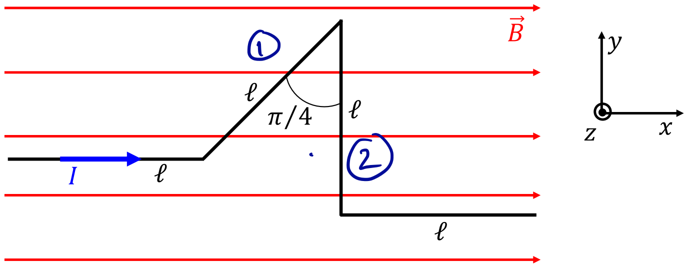
C. $(\frac{\sqrt{2}}{2})I\ell B_0\hat{z}$

D. 0

$$F_1 = -I\ell B \sin\frac{\pi}{4}\,\hat{z} = -I\ell B\frac{1}{\sqrt{2}}\,\hat{z}$$

$$F_2 = I\ell B\,\hat{z}$$

$$\bar{F}_{tot} = \left(1 - \frac{1}{\sqrt{2}}\right) I\ell B\,\hat{z}$$



Question 16

Two long parallel wires are hanging at 1 m distance, and the same current I is passed through the two wires along the same direction. The wires experience an attractive force per unit of length of $2 * 10^{-7}$ N/m. Evaluate I.

A. 0.5 A

B. 1 A

C. 2π A

D. 4π A

$$F = \frac{\mu_0 I^2}{2\pi r}$$

$$I^2 = \frac{F\,2\pi r}{\mu_0} = \frac{2\times10^{-7}\cdot 2\pi\cdot 1}{4\pi\times10^{-7}} = 1\,A$$

Question 17

A beam of unknown charged particles is accelerated with voltage V into a region with $\vec{B}$ field perpendicular to their trajectory, as shown in the figure below. A separation in space d at the end of the magnetic field region is measured when two different fields are used, $\vec{B}_1$ and $\vec{B}_2 = 2\vec{B}_1$. Determine the mass-to-charge ratio of the particles m/q as a function of B_1, d, V.

$$r = \frac{mv}{qB} \qquad r_2 = \frac{1}{2} r_1 \rightarrow d = 2\left(r_1 - \frac{1}{2} r_1\right)$$

$$d = r_1$$

A. $\frac{2V}{B_1^2 d}$

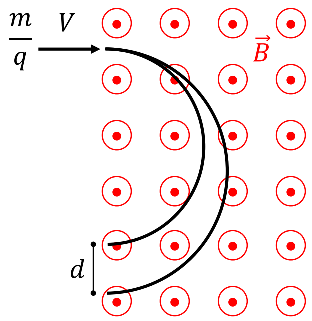
B. $\frac{B_1^2 d^2}{2V}$

C. $\frac{B_1 V}{2\pi d}$

D. $\frac{B_1 V^2}{4d}$

$$r_1 = \frac{m\sqrt{2V\frac{q}{m}}}{q\, B_1} = \sqrt{\frac{m}{q}}\,\frac{\sqrt{2V}}{B_1}$$

$$d^2 = \frac{m}{q}\,\frac{2V}{B_1^2}$$



$$\frac{m}{q} = \frac{d^2 B_1^2}{2V}$$

Question 18

Evaluate the modulus of the torque acting on a round loop of diameter d and with current I placed in an homogenous magnetic field $\vec{B}$ pointing at 60° away from the plane of the loop.

$$\vec{T} = \vec{m} \times \vec{B} \qquad |T| = mB\sin\theta$$

A. $\frac{\pi d^2 IB\sqrt{3}}{4}$

B. $\frac{\pi d^2 IB\sqrt{3}}{8}$

C. $\frac{\pi d^2 IB}{4}$

D. $\frac{\pi d^2 IB}{8}$

$$|T| = IAB\sin\theta = I\pi\left(\frac{d}{2}\right)^2 \sin 30^\circ B$$

$$|T| = \frac{d^2 I B \frac{1}{2}\pi}{4} = \frac{d^2 I B \pi}{8}$$

Question 19

The Earth's magnetic field points from South to North on the Earth's surface. Consider the metallic antenna on the roof of a car. In which direction is the car travelling if the potential induced by the Earth's magnetic field in the top of the antenna is higher than in the bottom?

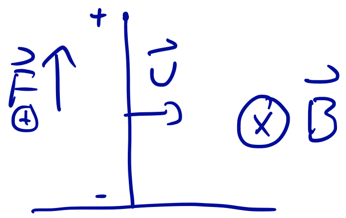
A. North

B. South $\Big\} \; V = 0$

C. East

D. West

$$\vec{F} = \vec{v} \times \vec{B}$$



Question 20

A current $I(t) = -\alpha t$ passes through a long straight wire, and a wire loop is placed in the same plane as the wire without touching it, as shown in the figure below. The loop is bent in such a way that it forms two squares of sides ℓ and at distance d from the straight wire, without touching itself in the intersection. Evaluate the voltage induced in the loop.

A. $\frac{\mu_0 \alpha \ell}{\pi} \ln\left(\frac{\ell+d}{d}\right)$

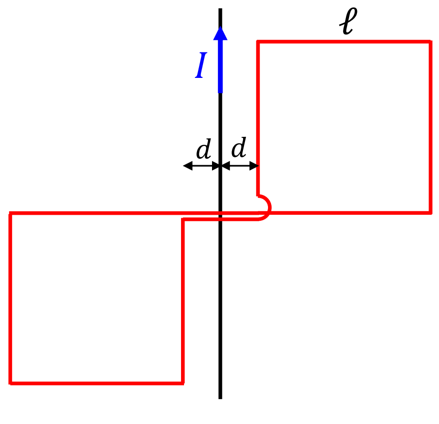
B. $\frac{\mu_0 \alpha \ell}{2\pi} \ln\left(\frac{\ell+d}{d}\right)$

C. $\frac{2\mu_0 \alpha \ell}{\pi} \ln\left(\frac{\ell+d}{d}\right)$

D. 0

$$B = \frac{\mu_0 I}{2\pi r} \qquad \Phi = \int \vec{B} \cdot d\vec{A}$$

$$\Phi = \int_d^{d+\ell} \frac{\mu_0 I \ell \, dr}{2\pi r} = \frac{\mu_0 I \ell}{2\pi} \int_d^{d+\ell} \frac{dr}{r} = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{d+\ell}{d}\right)$$



$$\Delta V = -2 \cdot \frac{d\Phi}{dt} = \frac{2\mu_0 \alpha \ell}{2\pi} \ln\left(\frac{\ell+d}{d}\right)$$

Question 21

Consider a straight solenoid of length ℓ and cross-section radius $a << \ell$ made of N windings. Because of a defect in the fabrication process, 1/4 of the windings randomly distributed are wound in the opposite direction. Evaluate the self-inductance of this solenoid.

A. $\frac{\mu_0 N^2 \pi a^2}{\ell}$

B. $\frac{3\mu_0 N^2 \pi a^2}{4\ell}$

C. $\frac{\mu_0 N^2 \pi a^2}{2\ell}$

D. $\frac{\mu_0 N^2 \pi a^2}{4\ell}$

$$L = \mu_0 n^2 \ell A$$

$$n = \frac{N'}{\ell} = \frac{N}{2\ell}$$

$$N' = \frac{3}{4}N - \frac{1}{4}N = \frac{1}{2}N$$

$$L = \frac{\mu_0 N^2 \pi a^2}{4\ell}$$

Question 22

The $B(H)$ plot of three different materials is shown in the figure below. Determine which one is paramagnetic, diamagnetic and ferromagnetic.

~~A.~~ 1) paramagnetic; 2) diamagnetic; 3) ferromagnetic

~~B.~~ 1) paramagnetic; 2) ferromagnetic; 3) diamagnetic

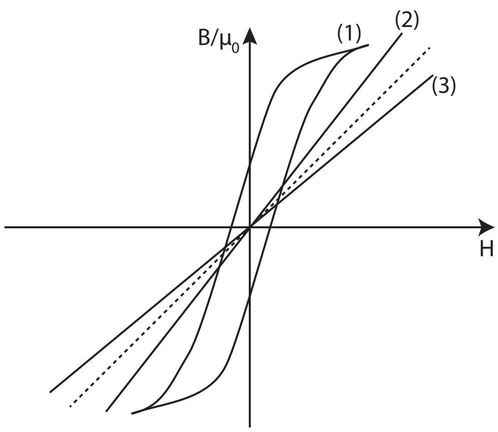
~~C.~~ 1) diamagnetic; 2) paramagnetic; 3) ferromagnetic

~~D.~~ 1) diamagnetic; 2) ferromagnetic; 3) paramagnetic

E. 1) ferromagnetic; 2) diamagnetic; 3) paramagnetic

F. 1) ferromagnetic; 2) paramagnetic; 3) diamagnetic $\rightarrow \mu_r < 1$

$\mu_r > 1$



Question 23

A cubic box made by mu-metal sheets of thickness d and relative permeability μ_r is placed in a homogeneous magnetic field B as shown in the figure below. Evaluate the maximum length ℓ of the cube edge that allows to completely screen the B field inside the box.

A. $d\mu_r B$

B. d/μ_r

C. $d/\mu_r B$

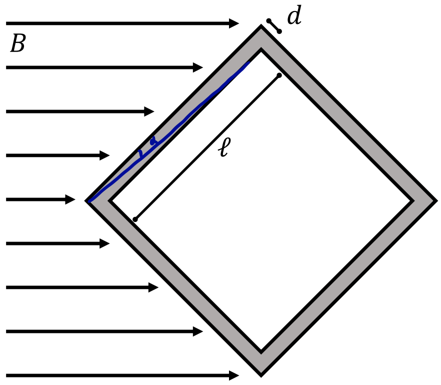
→ D. $d\mu_r$

$\tan\alpha = \frac{1}{\mu_r}$

$\tan\alpha = \frac{d}{\ell}$

$\frac{d}{\ell} = \frac{1}{\mu_r}$

$\ell = d\mu_r$



Question 24

Consider an oscillating electric dipole $p(t) = p_0 \sin(\omega t)$. What is the optimal length of an antenna emitting the same radiation as this dipole?

A. $\frac{4\pi c}{\omega}$

B. $\frac{2\pi c}{\omega}$

→ C. $\frac{\pi c}{\omega}$

D. $\frac{\pi c}{2\omega}$

$\lambda = \frac{2\pi c}{\omega}$

$\frac{1}{2}\lambda = \frac{\pi c}{\omega}$

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Problem 1

A solenoid made of N windings is wound around an iron core of relative permeability μ_r and shaped as a ring of radius b and circular cross-section of radius $a << b$.

a) Evaluate the self-inductance of the solenoid.

$$B = \mu_0 \mu_r n I \qquad n = \frac{N}{2\pi b} \qquad B = \frac{\mu_0 \mu_r N I}{2\pi b}$$

$$\Phi = B \pi a^2 N = \frac{\mu_0 \mu_r N^2 a^2 I}{2b}$$

$$L = \frac{\Phi}{I} = \frac{\mu_0 \mu_r N^2 a^2}{2b}$$

$$\boxed{L = \frac{\mu_r \mu_0 N^2 a^2}{2b}}$$

A current I_0 flows in the solenoid since it is connected to a potential V_0. An electric switch instantaneously disconnects the voltage and shorts the solenoid circuit at $t = 0$.

b) Determine the evolution in time of the current in the solenoid $I(t)$.

$$V = -L \frac{dI}{dt} \qquad R = \frac{V_0}{I_0} \qquad V = IR = \frac{I V_0}{I_0}$$

$$I = -\frac{L}{R} \frac{dI}{dt} \qquad \frac{dI}{I} = -\frac{R\,dt}{L} \qquad t=0:\ I = I_0$$

$$I(t) = I_0 e^{-\frac{R}{L} t}$$

$$\frac{R}{L} = \frac{2b V_0}{\mu_r \mu_0 N^2 a^2 I_0}$$

$$\boxed{I(t) = I_0 e^{-\left[\frac{2 b V_0 t}{\mu_r \mu_0 N^2 a^2 I_0}\right]}}$$

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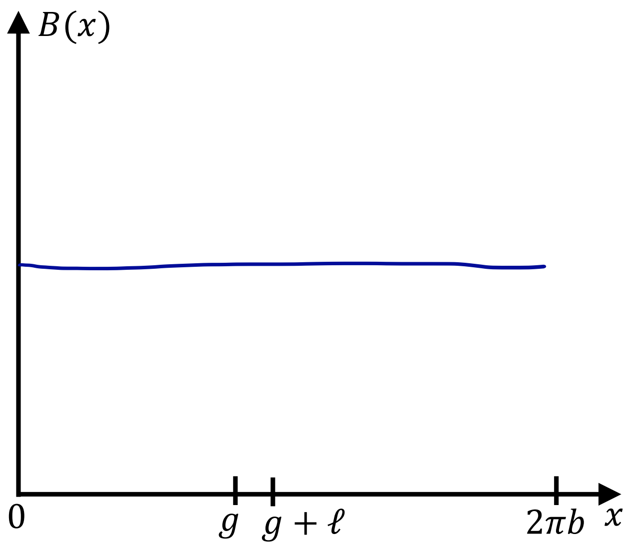
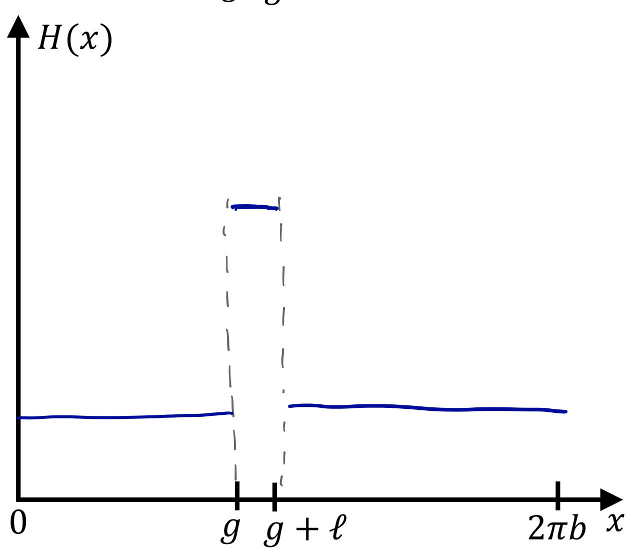
Consider now the case where the solenoid is connected again to the source V_0, but the iron core has a small gap of size $\ell << b$ in air ($\mu_{r_{air}} \approx 1$). Since the gap is small, you can consider that the magnetic field lines follow the same path in the gap as if they were in the iron material.

c) Make a plot of the B and H fields along the circular path of length $2\pi b$ inside the solenoid, where g and $g + \ell$ in the figures below delimit the position of the gap.





$H = \frac{B}{\mu_0 \mu_r}$

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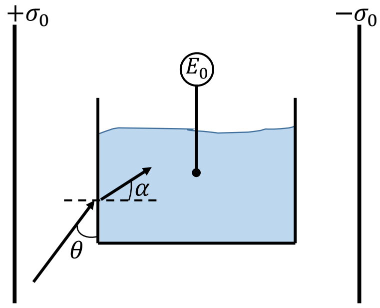
Problem 2

A beaker of negligible width is filled with a non-conductive liquid and placed in the space inside a parallel plate capacitor charged with surface charge density σ_0, as shown in the figure below. With a device, an electric field E_0 is probed inside the liquid.



a) Evaluate the relative permittivity of the liquid.

$$D = \sigma_0 \qquad E_0 = \frac{D}{\varepsilon_0 \varepsilon_r}$$

$$\varepsilon_r = \frac{\sigma_0}{\varepsilon_0 E}$$

$$\boxed{\epsilon_r = \frac{\sigma_0}{\varepsilon_0 E_0}}$$

A laser beam is sent through the beaker with an angle $\theta = 60°$ as shown in the figure.

b) Evaluate the angle α.

$$n = \sqrt{\varepsilon_r} \qquad n \sin\alpha = \sin 30° = 1/2$$

$$\alpha = \arcsin\left(\frac{1}{2\sqrt{\varepsilon_r}}\right)$$

$$\boxed{\alpha = \arcsin\left(\sqrt{\frac{\varepsilon_0 E_0}{4\sigma_0}}\right)}$$

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Cherenkov radiation occurs when the velocity of a charged particle exceeds the speed of light.

c) Ignoring relativistic effects, what energy should an electron in the liquid have to emit Cherenkov radiation?

$$c_m = \frac{c}{\sqrt{\varepsilon_r}}$$

$$E_k = \frac{1}{2} m_e v^2 > \frac{1}{2} m_e \frac{c^2}{\varepsilon_r} = \frac{1}{2} m_e c^2 \frac{\varepsilon_0 E_0}{\sigma_0}$$

$$\boxed{E_k > \frac{1}{2} m_e c^2 \frac{\varepsilon_0 E_0}{\sigma_0}}$$